Quiz 3, Linear Algebra

Dr. Adam Graham-Squire

Name: ___________________________________

1. (4 points) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that first reflects points through the vertical x_2-axis (or y-axis) and then rotates points $\frac{\pi}{2}$ radians counterclockwise. Find the standard matrix of T. Show your work!

2. (3 points) Given $A = \begin{bmatrix} 5 & 3 & 2 \\ -4 & 1 & -5 \\ -4 & -1 & -3 \\ 1 & 0 & 1 \end{bmatrix}$, observe that the first column is the sum of the second and third columns. Without performing any row operations to reduce the matrix, find a nontrivial solution to $A\mathbf{x} = \mathbf{0}$. [Hint: Write $A\mathbf{x} = \mathbf{0}$ as a vector equation.]

3. (3 points) Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation, and let $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ be a linearly dependent set in $\mathbb{R}^n$. Explain why the set $\{T(\mathbf{v}_1), T(\mathbf{v}_2), T(\mathbf{v}_3)\}$ is linearly dependent. (Recall: a set $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ is linearly dependent if there exist $c_1, c_2, \ldots, c_n$, not all zero, such that $c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \cdots + c_n\mathbf{v}_n = \mathbf{0}$).